

## Category Theory

### Exercise Sheet 11

**Exercise 1.** Show that the forgetful functor from the category of fields to the category of rings admits neither a left nor a right adjoint.

**Exercise 2.** Use the adjoint functor theorem to prove the existence of free groups.

**Exercise 3.** Let  $C$  be a locally small category which admits all small coproducts. Show that a functor  $F: C \rightarrow \mathbf{Set}$  is representable if and only if it admits a left adjoint.